Ma 3b Practical – Recitation 3

February 27, 2025

Warm up (see below): recall definitions of pmf, cdf, joint distribution, $E(X)$, $Var(X)$

Exercise 1. (discrete pmf, cdf) Consider throwing two coins and let the random variable X be the number of coins that face upward. What is pmf? What is cdf?

Exercise 2. (joint distribution) Consider throwing two dice. Let 1_{X_1} be the random variable that takes value 1 if X_1 is even, and take value 0 if X_1 is odd and let 1_{X_2} be the random variable that takes value 1 if X_2 is even, and takes value 0 if X_2 is odd. What is the joint cumutative distribution function 1_{x_1} and 1_{x_2}?

Exercise 3. As in Exercise 2, compute $E(1_{x_1})$ and the following expectations

1. What is $E(1_{x_1} \cdot 1_{x_2})$? Is it true that $E(1_{x_1} \cdot 1_{x_2}) = E(1_{x_1}) \cdot E(1_{x_2})$?
2. What is $E(1_{x_1}^2)$? Is it true that $(E(1_{x_1}))^2 = E(1_{x_1}^2)$?

Exercise 4. (optional) If X_1, X_2 are two independent discrete random variables, prove that $E(X_1X_2) = E(X_1)E(X_2)$. If X_1, X_2 are not independent, give a counterexapmle. (Exercise 3)

Exercise 5. (continuous pmf, cdf) Let X be a continuous random variable with pmf given by

$$f(x) = \begin{cases} x, & \text{for } 0 \leqslant x \leqslant 1 \\ 2-x, & \text{for } 1 < x \leqslant 2 \\ 0, & \text{otherwise} \end{cases}.$$

1. Find cdf $F(0,5)$ and $F(1.5)$
2. Find the expectation value of x.

Exercise 6. (optional: linearity of expectation) Assume there are n types of items in the supermarket, and each time, I buy any one of them randomly. What is the expected number of times I need to go to the supermarket, to have at least one from each type? For n=365, we can conclude on the number of friends we need to have, such that we will have birthday party everyday.

Definition 1 *The probability mass function is given by:*

$$P(X = x) := P(\{\omega \in \Omega \,|\, X(\omega) = x\}) = p_X(x)$$

Definition 2 *The cumulative mass function is given by:*

$$F_X(x) = P(X \leqslant x)$$

Definition 3 *The joint distribution for discrete random variables* x, y *is given by:*

$$p_{X,Y}(x, y) = P(X = x, Y = y). \tag{1}$$

Definition 4 *The expectation value is given by:*

$$E(X) = \sum_{i=1}^{n} x_i p_i$$

That is, the expectation is a weighted sum of the values (each x_i*) weighted by their probabilities (each* p_i*).*

1 Solutions:

Exercise 6, see slack.